# An overview of variational damage and fracture

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collaborators at various stages:

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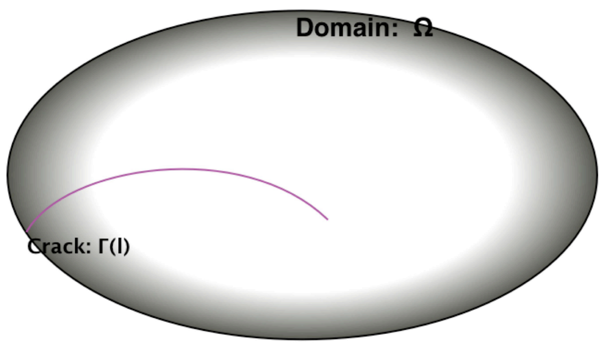
## Quasi-static Brittle Fracture



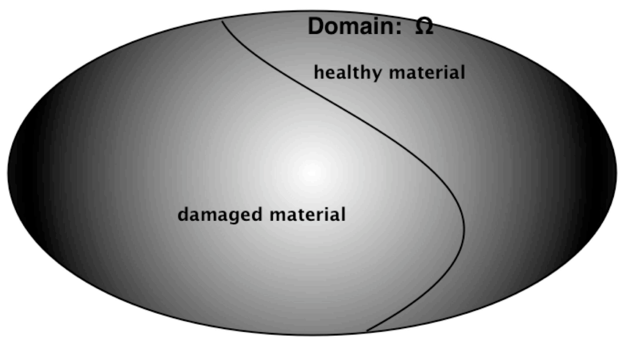

$W$ elastic energy density/ $\mathcal{L}(t,.)$ represents the applied loads / $g(t)$ the displacement b.c.'s

$\Gamma(l) \subset \hat{\Gamma} \subset \overline{\Omega}$: the crack of length $l$ (following a preset path $\hat{\Gamma}$)

- At fixed $l$, $u(l)$ min.: $E^{el}(l;v) := \int_{\Omega\setminus\Gamma(l)} W(e(v))\,dx - \mathcal{L}(t,v)$ among all $v$'s with $v = g(t)$ on $\partial\Omega$: $P(l;t) :=$ value at minimum
- The energy release rate is: $G(l;t) := -\dfrac{\partial P}{\partial l}(l;t)$
- Griffith: $G(l(t);t) \leq k$ / $\dot{l} \geq 0$ / $(G(l(t);t) - k)\,\dot{l}(t) = 0$

$\Rightarrow$ gives $l(t)$ (sometimes...)

## Quasi-static Brittle Partial Damage




same as for fracture

Resp. elast. $A_d$ (damaged), $A_h$ (healthy): $\chi$: characteristic fctn.
damaged state/ energy dens. : $W(\chi; e) = \frac{1}{2}(\chi A_d + (1-\chi)A_h)e.e$

- At fixed $\chi$, $u(\chi)$ minimizes: $E^{el}(\chi; v) := \int_\Omega W(\chi; e(v))\, dx - \mathcal{L}(t, v)$ among all $v$'s with $v = g(t)$ on $\partial\Omega$:
- "Griffith": $-\dfrac{\partial W}{\partial \chi}(u(\chi(x,t);x)) \le k$ / $\dot{\chi} \ge 0$ /
$\left(-\dfrac{\partial W}{\partial \chi}(u(\chi(x,t);x) - k\right) \dot{\chi}(x,t) = 0 \Rightarrow$ gives $\chi(t)$ (never!)

**Main Goals**

First and foremost:

Prediction of crack path and evolution/ growth of damaged zone

Then, maybe:

Initiation

## Variational reformulation

$u$: displacement / $d$: internal variable $= l$, or $\chi$ / $\mathcal{D}$: associated energy –

1-homogeneous in $d$: $kd$ for fracture and $k\int_\Omega d(x)\,dx$ for damage

In 2 cases, "classical" model $\Leftrightarrow$ three-pronged formulation at time $t$:

1. local stability: $(u(t), d(t))$ is a local minimizer for:
   $E^{el}(d,v) + \mathcal{D}(d)$ among all: $v = g(t)$ on $\partial\Omega,\ d \geq d(t)$

   (where $\geq$ is the relevant ordering relation on the internal variable $d$)

2. irreversibility: $d(t)\nearrow$ with $t$ (for that order)

3. energy balance: $\dfrac{d}{dt}\left(E^{el}(d(t), u(t)) + \mathcal{D}(d(t))\right) =$ power of surface forces generated by $g(t)$ + $\dot{\mathcal{L}}(u(t))$.

## Global minimization

Our original departure: Replace the first item by a criterion of global minimality $\Rightarrow$ Fracture: $\Gamma(l)$ becomes any set $\Gamma$ with finite "length"

$\Downarrow$

A whole bunch of results starting with F.-Marigo 93, Ambrosio-Braides 95, F.-Marigo 98, Dal Maso-Toader 02, F.-Larsen 03,........

Not OK for fracture / Not OK for damage

because, among other things,

$\swarrow$ $\searrow$

. Forces drive minimum to $-\infty$

. Energetic landscape too bumpy

. Internal issue: how to compute globally min. evns? Bourdin 07

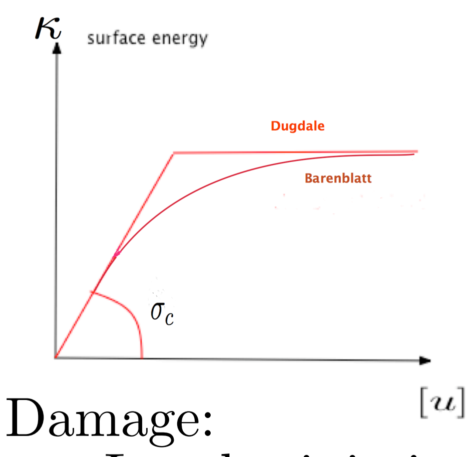
. Min. process $\Rightarrow$ mix of healthy and damaged mat.: is relaxed evolution a "good" appr. of some classical evolution?

. Computational issues

**Is local min. the solution?** **Or are we missing some physics?**

Fracture:

- No existence (other than global min.)
- Generically, no initiation Chambolle-Giacomini-Ponsiglione 08
- Does not cure forces
- Allow for energ. increase: $\epsilon$-slides Larsen ?
- The role of viscosity? Truskinovsky
- Switch to dift. energy: cohesive type ⇓




. introduces relaxation Braides-Dal Maso-Garroni 99 $\longrightarrow$ plasticity

. not clear that no Cantor part

. no evolution theory, except for pre-set paths Dal Maso-Zanini 07

Damage:

- Local minimizers are global minimizers Garroni-Larsen 08
- Damage regularization Benallal

## Outstanding issues

Fracture:

- Kinking: Oleaga, new results pending
- Non-interpenetration
- Dead forces Chambolle

Fracture and Damage: the import(ance) of dynamics
Combescure, Marder, Marigo

Damage to Fracture, or Fracture to Damage?

and then there is Plasticity Mora